CORRIGE

- **Ces éléments de correction n'ont qu'une valeur indicative. Ils ne peuvent en aucun cas engager la responsabilité des autorités académiques, chaque jury est souverain.**

EPREUVE E4 : ETUDE DES CONSTRUCTIONS

Année 2006

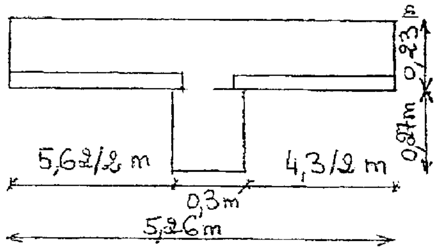
Sous-épreuve : U 41 BTS BATIMENT

Elaboration d'une note de calcul de structures

CORRECTION

ETUDE A :

A_1_ On lit : $L_{14} = 6{,}96\,m$



La poutre 14 reprend :

C.P. : kN/m

pp retombée poutre : $25 \times 0{,}27 \times 0{,}3 = 2{,}025$

pp dalle : $25 \times 0{,}23 \times 5{,}26 = 30{,}245$

p terre étanchéité : $5{,}85 \times 5{,}26 = 30{,}771$

$\underline{\underline{g_{14} = 63{,}041\ kN/m}}$

C.E. : $1{,}5 \times 5{,}26 = \underline{\underline{7{,}890\ kN/m = q_{14}}}$

A_2_ B6.2.210 du BAEL

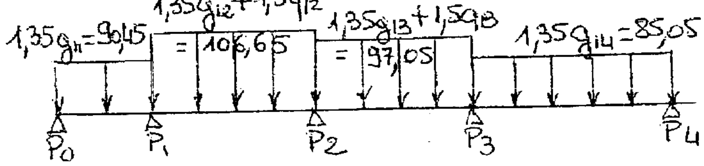
- $q_{11} \leq 2g_{11}$, $q_{12} \leq 2g_{12}$, $q_{13} \leq 2g_{13}$, $q_{14} \leq 2g_{14}$, et $q = 1{,}5\ kN/m^2 < 5\ kN/m^2$ OK
- $b_0 = 0{,}3m$ et $h = 0{,}5m$ partout constants OK
- fissuration peu préjudiciable OK
- mais $\dfrac{L_{11}}{L_{12}} = \dfrac{3{,}185}{5{,}565} = 0{,}572 < 0{,}8$ ~~OK~~

Une des conditions n'est pas vérifiée. Il faut utiliser la méthode Caquot minorée. On utilisera 1 comme coefficient des moments dus aux charges permanentes.

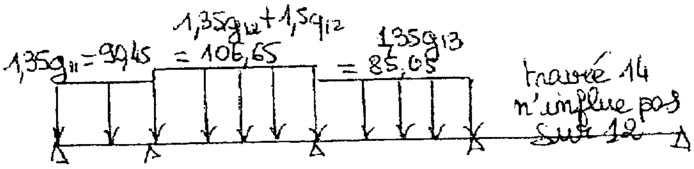
A_3_ moment maximal en

charges en kN/m

a) P_2



b) travée 12



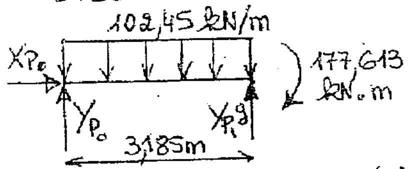
A-4- $p_{11} = 1{,}35\, g_{11} + 1{,}5\, q_{11} = 102{,}45$ kN/m $\qquad p_{12} = 1{,}35\, g_{12} = 93{,}15$ kN/m

$L'_{11} = L_{11} = 3{,}185$ m $\qquad L'_{12} = 0{,}8\, L_{12} = 0{,}8 \times 5{,}565 = 4{,}452$ m

supprimé.

$$|M_{P_1}| = \frac{102{,}45 \times 3{,}185^3 + 93{,}15 \times 4{,}452^3}{8{,}5 \times (3{,}185 + 4{,}452)} = 177{,}613 \text{ kN.m}$$

Isolons la travée 11 :



$$\begin{cases} X_{P_0} = 0 \\ Y_{P_0} + Y_{P_1}^g - 102{,}45 \times 3{,}185 = 0 \\ Y_{P_1}^g \times 3{,}185 - 177{,}613 - 102{,}45 \times \frac{3{,}185^2}{2} = 0 \end{cases}$$

d'où, $Y_{P_1}^g = 218{,}92$ kN et $Y_{P_0} = 107{,}39$ kN

$x_{max} = \frac{107{,}39}{102{,}45} = 1{,}048$ m $\Rightarrow M_{max\ \text{travée } 11} = \frac{107{,}39 \times 1{,}048}{2} = 56{,}28$ kN.m

A-5-1- F.P.P. $\Rightarrow$ calcul aux E.L.U.
sur appui $\Rightarrow$ calcul en section rectangulaire. $b_0 = 0{,}3$ m

$f_{bu} = \frac{0{,}85 f_{cj}}{\theta \gamma_b} = 14{,}17$ MPa $\qquad f_{su} = \frac{f_e}{\gamma_s} = 434{,}8$ MPa

$\mu_u = \frac{0{,}196}{0{,}3 \times 0{,}45^2 \times 14{,}17} = 0{,}228 > 0{,}186$ pivot B
$< \mu_{lu} = 0{,}260$ pas d'aciers comprimés

$\alpha_u = 0{,}328 \qquad z_u = 0{,}391$ m

$A_{calc} = \frac{0{,}196}{434{,}8 \times 0{,}391} = 11{,}53$ cm² $\qquad A_{min} = 1{,}3$ cm²

choix : 3 HA 16 + 3 HA 16 ($A_{réelle} = 12{,}06$ cm²)

$d_{réelle} = 50 - 2 - 0{,}8 - 1{,}6 = 45{,}6$ cm > 45 cm OK

A-§-2 $\bar{\tau}_u = \min\left(\frac{0{,}2 f_{cj}}{\gamma_b} ; 5\text{ MPa}\right) = 3{,}33\text{ MPa}$

$\tau_u = \frac{|V_u|}{b_o\, d} = \frac{0{,}225}{0{,}3 \times 0{,}45} = 1{,}67\text{ MPa} \leq \bar{\tau}_u$ On peut mettre des armatures d'âmes droites.

Pour entourer 2 lits de 3 HA16, choisissons 1 cadre + 1 étrier.

si $\phi_t = 6$ mm, $\underline{\underline{A_t}} = 4 \times \frac{\pi \times 0{,}006^2}{4} = \underline{\underline{1{,}131\text{ cm}^2}}$

$\underline{\underline{St_{max}}} = \min\left(0{,}9 \times 0{,}45 ; 40\text{ cm} ; \frac{1{,}131.10^{-4} \times 500}{0{,}4 \times 0{,}3}\right) = \underline{\underline{40\text{ cm}}}$

$k = 0$ car reprise de bétonnage sans traitement

$St_o \leq \frac{0{,}9 \times 1{,}131.10^{-4} \times 434{,}8}{0{,}3 \times 1{,}67} = 0{,}088\text{ m} \Rightarrow \underline{\underline{St_o = 8\text{ cm}}}$

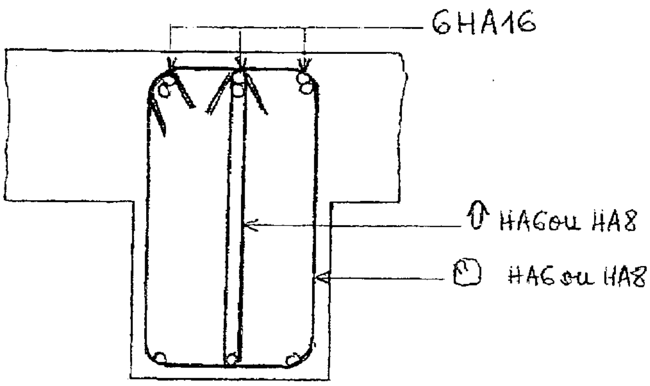
1er lit à 4 cm du nu d'appui.

si $\phi_t = 8$ mm, $\underline{\underline{A_t}} = 4 \times \frac{\pi \times 0{,}008^2}{4} = \underline{\underline{2{,}011\text{ cm}^2}}$

$\underline{\underline{St_{max}}} = \min\left(0{,}9 \times 0{,}45 ; 40\text{ cm}, \frac{2{,}011.10^{-4} \times 500}{0{,}4 \times 0{,}3}\right) = \underline{\underline{40\text{ cm}}}$

$St_o \leq \frac{0{,}9 \times 2{,}011.10^{-4} \times 434{,}8}{0{,}3 \times 1{,}67} = 0{,}157\text{ m} \Rightarrow \underline{\underline{St_o = 13\text{ cm}}}$

A-§-3-



ETUDE B :

B_1_ $\lambda = \frac{\sqrt{12}\, Lf}{a} = \frac{\sqrt{12} \times 3}{0,2} = 51,96 > 50$ et $\lambda < 70$

$\alpha = 0,6 \times \left(\frac{50}{51,96}\right)^2 = 0,556$

$Br = (0,2-0,02) \times (0,4-0,02) = 0,0684\ m^2$

$A_{th} = \left[\frac{Nu}{\alpha} - \frac{Br \times fc28}{0,9\gamma b}\right] \times \frac{1}{fsu} = \left[\frac{0,558}{0,556} - \frac{0,0684 \times 25}{0,9 \times 1,5}\right] \times \frac{1}{434,8} = -6,05.10^{-4} m^2 < 0$

On met la section d'armatures minimale.

$A_{min1} = 4u = 4 \times (0,2 + 0,4) \times 2 = 4,8\ cm^2$

$A_{min2} = 0,2\%B = 0,2\% \times 0,2 \times 0,4 = 1,6\ cm^2$

$\underline{\underline{A = 4,8\ cm^2}}$

$s_l \leq min(a+10cm ; 40\ cm) = 30\ cm$. Il faut donc 6 aciers.

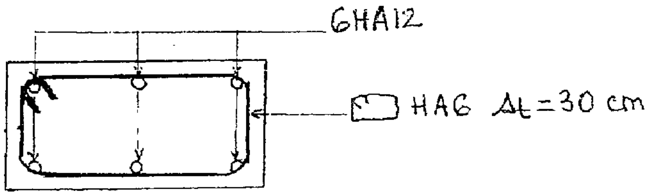
Choix : 6 HA 12 ($6,79\ cm^2$)

B_2_ $\phi_t = \frac{\phi_l}{3} = \frac{12}{3} = 4\ mm \Rightarrow \underline{\phi_t = 6\ mm}$

$s_t \leq min(40\ cm ; a+10\ cm ; \cancel{15\phi_l}) = 30\ cm \Rightarrow \underline{s_t = 30\ cm}$

$\phi_l < 20\ mm$ et $A = A_{min} \Rightarrow$ pas besoin d'épingle.

B_3_



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ETUDE C :

C-1 – $A \geqslant \sqrt{\dfrac{N_u \times a}{b \times q}} = \sqrt{\dfrac{0{,}558 \times 0{,}2}{0{,}4 \times 0{,}3}} = 0{,}964\,m$ On choisit : $A = 1m$

$B \geqslant \sqrt{\dfrac{N_u \times b}{a \times q}} = \sqrt{\dfrac{0{,}558 \times 0{,}4}{0{,}2 \times 0{,}3}} = 1{,}93\,m$ On choisit : $B = 2\,m$

$d \geqslant \dfrac{B-b}{4} = \dfrac{2-0{,}4}{4} = 0{,}4\,m \Rightarrow h = 0{,}45\,m$

$P_{semelle\,u} = 1{,}35 \times 0{,}025 \times 1 \times 2 \times 0{,}45 = 0{,}03\,MN.$

$\dfrac{0{,}558 + 0{,}03}{1 \times 2} = 0{,}294\ MPa \leqslant q = 0{,}3\,MPa$ OK

Ces dimensions conviennent.

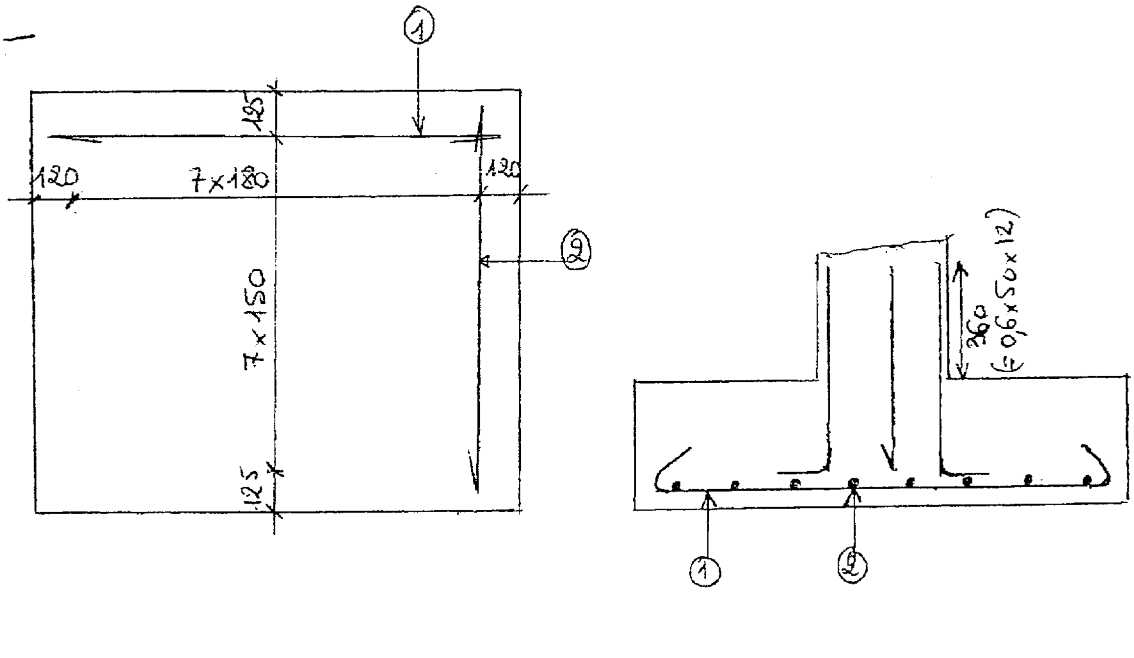
C-2 – $A_1 = \dfrac{1{,}1 \times N_u \times (B-b)}{8 \cdot d_1 \cdot f_{su}} = \dfrac{1{,}1 \times 0{,}558 \times (1{,}50 - 0{,}4)}{8 \times 0{,}34 \times 434{,}8} = 5{,}71\ cm^2$

① choix : 8 HA 10 (e = 15 cm) ($6{,}28\ cm^2$)

$A_2 = \dfrac{1{,}1 \times N_u \times (A-a)}{8 \cdot d_2 \cdot f_{su}} = \dfrac{1{,}1 \times 0{,}558 \times (1{,}3 - 0{,}2)}{8 \times 0{,}33 \times 434{,}8} = 5{,}88\ cm^2$

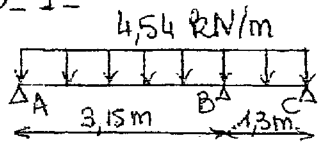
② choix : 8 HA10 (e = 18 cm)

C-3 –



ETUDE D :

D_1_



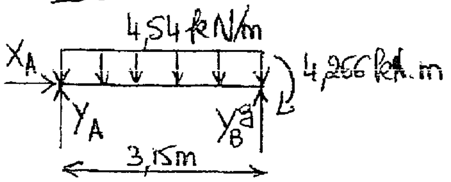
Appliquons le théorème des 3 moments :

$M_A = M_C = 0$ car appuis de rive

$$2 \times (3{,}15 + 1{,}3)\ M_B = 6EI\left[-\frac{4{,}54 \times 1{,}3^3}{24\ EI} - \frac{4{,}54 \times 3{,}15^3}{24\ EI}\right]$$

d'où, $\underline{\underline{M_B = -4{,}266\ kN.m}}$

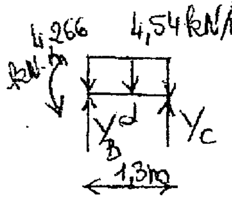
Isolons la travée 1 :



$$\begin{cases} X_A = 0 \\ Y_A + Y_B^g - 4{,}54 \times 3{,}15 = 0 \\ Y_B^g \times 3{,}15 - 4{,}54 \times \dfrac{3{,}15^2}{2} - 4{,}266 = 0 \end{cases}$$

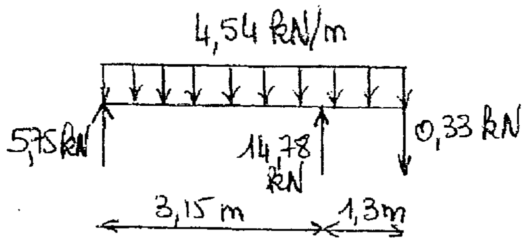
$\underline{Y_B^g = 8{,}55\ kN}$ $\underline{Y_A = 5{,}75\ kN}$

Isolons la travée 2 :

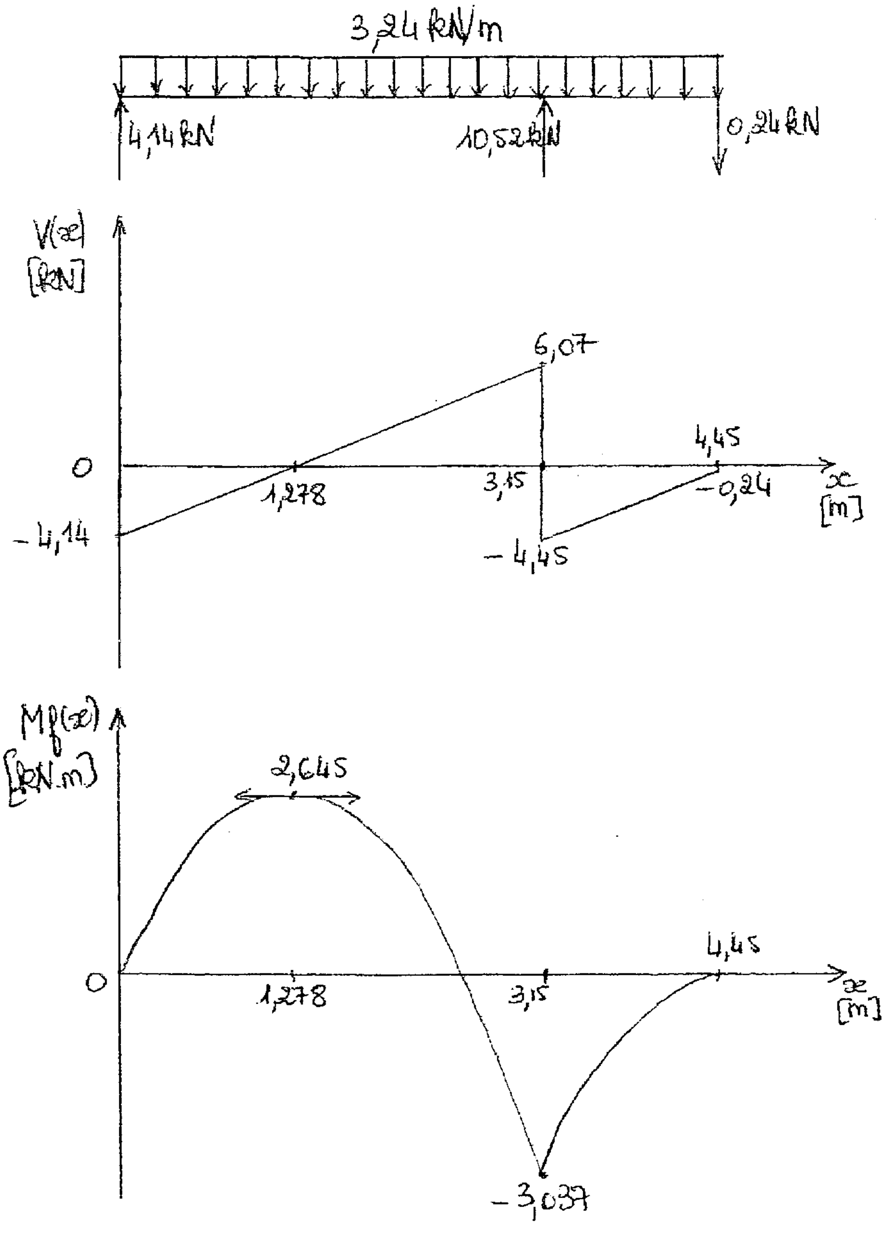


$$\begin{cases} Y_B^d + Y_c - 4{,}54 \times 1{,}3 = 0 \\ Y_c \times 1{,}3 - 4{,}54 \times \dfrac{1{,}3^2}{2} + 4{,}266 = 0 \end{cases}$$

$\underline{Y_c = -0{,}33\ kN}$ $\underline{Y_B^d = 6{,}23\ kN}$



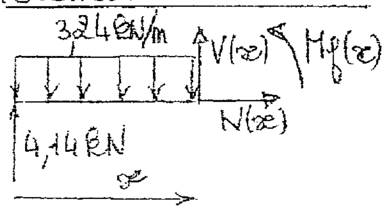
D-2-



3,24 kN/m
4,14 kN
10,52 kN
0,24 kN
V(x)
[kN]
6,07
4,45
0
1,278
3,15
-0,24
x
[m]
-4,14
-4,45
Mf(x)
[kN.m]
2,645
0
1,278
3,15
4,45
x
[m]
-3,037

D_3_ $E = 11\,600$ MPa $I = \frac{bh^3}{12} = \frac{0{,}063 \times 0{,}175^3}{12} = 2{,}81 \cdot 10^{-5}\,m^4$

Première méthode : $y''(x) = \frac{Mf(x)}{EI}$



dans la première travée,

$Mf(x) = -3{,}24\frac{x^2}{2} + 4{,}14\,x$

$EI\,y_1''(x) = -1{,}62\,x^2 + 4{,}14\,x$

$EI\,y_1'(x) = -1{,}62\frac{x^3}{3} + 4{,}14\frac{x^2}{2} + k_1$

$EI\,y_1(x) = -1{,}62\frac{x^4}{12} + 4{,}14\frac{x^3}{6} + k_1 x + k_2$

conditions aux limites :

$y_1(0) = 0 = k_2$ et $y_1(3{,}15\,m) = 0$

$EI\,y_1(3{,}15m) = -1{,}62 \times \frac{3{,}15^4}{12} + 4{,}14 \times \frac{3{,}15^3}{6} + k_1 \times 3{,}15 = 0$

d'où, $k_1 = -2{,}627$ d'où $y_1(x) = \frac{1}{EI_1}\left[-1{,}62\frac{x^4}{12} + 4{,}14\frac{x^3}{6} - 2{,}627x\right]$

au milieu de la travée

$x = \frac{3{,}15}{2} = 1{,}575\,m$

$y_1(1{,}575m) = \frac{10^{-3}}{11600 \times 2{,}81 \cdot 10^{-5}}\left[-1{,}62 \times \frac{1{,}575^4}{12} + 4{,}14 \times \frac{1{,}575^3}{6} - 2{,}627 \times 1{,}575\right]$

$y_1 = -6{,}97$ mm en $x = 1{,}575\,m$ $Y_1 = 7{,}0$ mm

NB : La flèche maximale se trouve en $x = 1{,}42\,m$, valeur qui annule $y_1'(x)$, $y_1(1{,}42m) = -7{,}07$ mm, valeur proche de celle obtenue.

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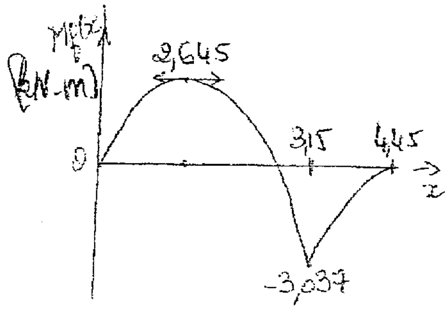
Seconde méthode :



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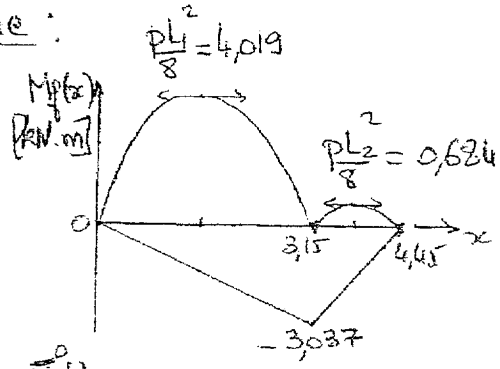

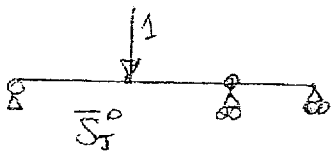

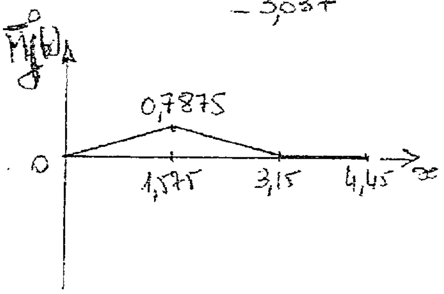

$$\Delta_j = \frac{1}{EI} \int_{\text{structure}} M(x)\, \overline{M}_j^0(x)\, dx$$

$$\Delta_j = \frac{10^{-3}}{11600 \times 2{,}81 \cdot 10^{-5}} \times \left[\int_0^{3{,}15\,m} 4{,}019 \times 0{,}7875 \, dx + \int_0^{3{,}15\,m} -3{,}037 \times 0{,}7875 \, dx \right]$$

$$\Delta_j = \frac{10^{-3}}{11600 \times 2{,}81 \cdot 10^{-5}} \times \left(\frac{5}{12} \times 3{,}15 \times 4{,}019 \times 0{,}7875 + \frac{3{,}15}{4} \times (-3{,}037) \times 0{,}7875 \right)$$

$$\underline{\underline{\Delta_j = 6{,}966 \cdot 10^{-3}\,m = 7{,}0\ mm}}$$

D-4-1- $\frac{I}{v} = W = \frac{bh^2}{6} = \frac{0{,}063 \times 0{,}175^2}{6} = 3{,}216 \cdot 10^{-4} m^3$

$f_{m,d} = k_{mod} \times \frac{f_{mk}}{\gamma_M} = 0{,}9 \times \frac{24}{1{,}25} = 17{,}28$ MPa

$|Mf_{max}| = 4{,}27$ kN.m

$|\sigma_{m,d}| = \frac{|Mf_{max}|}{W} = \frac{4{,}27 \cdot 10^{-3}}{3{,}216 \cdot 10^{-4}} = 13{,}277$ MPa

critère de résistance : $\frac{|\sigma_{m,d}|}{kh \times f_{m,d}} = \frac{13{,}277}{1{,}15 \times 17{,}28} = \underline{0{,}668 \leq 1}$ OK

D-4-2- $|V_{max}| = 8{,}5$ kN

$\tau_d = \frac{3|V_{max}|}{2A} = \frac{3 \times 8{,}5 \cdot 10^{-3}}{2 \times 0{,}063 \times 0{,}175} = 1{,}156$ MPa

$f_{v,d} = k_{mod} \times \frac{f_{v,k}}{\gamma_M} = 0{,}9 \times \frac{2{,}7}{1{,}25} = 1{,}944$ MPa

critère de résistance : $\frac{\tau_d}{f_{v,d}} = \underline{0{,}595 \leq 1}$ OK

D-4-3-

$u_{net,inst} = u_{g,inst} + u_{q,inst} - u_0$
$= 5 + 2{,}6 - 0$

$u_{net,inst} = 7{,}6$ mm

$u_{net,fin} = u_{net,inst}(g) \times (1 + k_{def}) + u_{net,inst}(s) = 5 \times (1 + 0{,}6) + 2{,}6 = \underline{10{,}6 \text{ mm}}$

conditions de flèche :

$u_{net,inst} = \underline{7{,}6 \text{ mm}} \leq \frac{L}{300} = \frac{3150}{300} = \underline{10{,}5 \text{ mm}}$ OK

$u_{net,fin} = \underline{10{,}6 \text{ mm}} \leq \frac{L}{250} = \frac{3150}{250} = \underline{12{,}6 \text{ mm}}$ OK